Recent pQCD Results on Heavy Quark Production

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Motivation

Good reasons to focus on heavy quarks:

- Heavy quarks introduce a new mass scale relative to which QGP properties can be constrained

$$T / m, \;\; \mu_D / m$$

- Due to the heavy quark mass are reliably computable in pQCD (?)
- Energy loss of heavy quarks should be a test of the pQCD energy loss mechanism (?)

....

Should go back to the very basic perturbative results in p+p and p+A

Outline of the Discussion

- **Heavy quark (HQ) production in p+p:**
  - Alphas, powers, logarithms and phenomehology

Based on: I.V., M.B.Johnson, T.Goldman, J.W.Qiu; hep-ph/0605200

  - Detailed analysis of hard scattering in LO
  - New results for back-to-back heavy quark correlations

- **Heavy quarks shadowing in p(d)+A:**
  - Calculations of shadowing in the DIS
  - Calculations in p+A reactions for light and heavy quarks

- **Energy loss and suppression of HQ in p(d)+A:**
  - Connecting the low energy with the high energy data
  - Implications for heavy quarks in (p)d+A

Based on: I.V., in preparation

- **New solutions for initial state E-loss:**
  - Analytic and numerical results, phenomenology

Classification of pQCD Calculations

LO, NLO, NNLO expansion

$$\frac{d\sigma}{dp_T^2} = A(m)\alpha_s^2 + B(m)\alpha_s^3 +$$

LL, NLL, NNLL expansion

$$\left(\alpha_s^2 \sum_{i=2}^{\infty} a_i (\alpha_s \log \mu/m)^i + \alpha_s^3 \sum_{i=1}^{\infty} b_i (\alpha_s \log \mu/m)^i \right) \times G(m,p_T) \to 1, \quad \frac{m}{p_T} \to 0$$

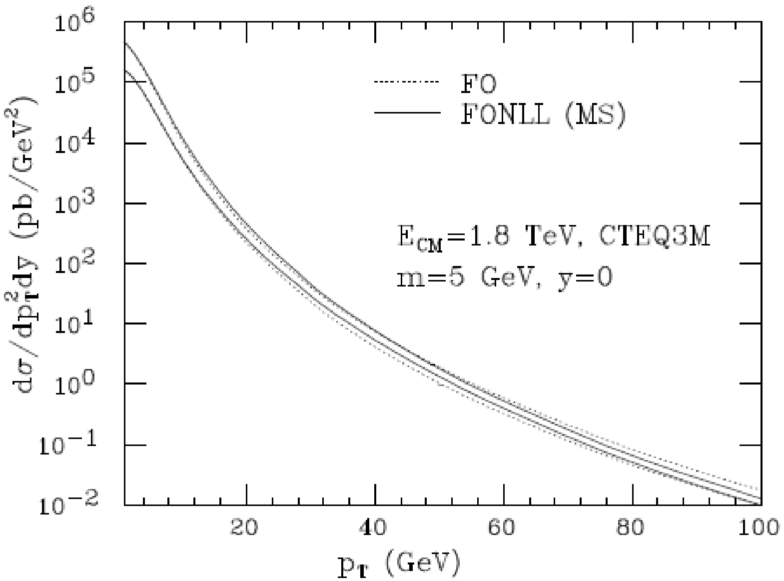
$$+\mathcal{O}(\alpha_s^4 (\alpha_s \log \mu/m)^i) + \mathcal{O}(\alpha_s^4 \times \mathrm{PST}) \,,$$

m/p_T, $(m/p_T)^2$ power corrections

Schematic NLO and NNL

$A(m),\ B(m)$ – coefficient functions

Will return to power corrections

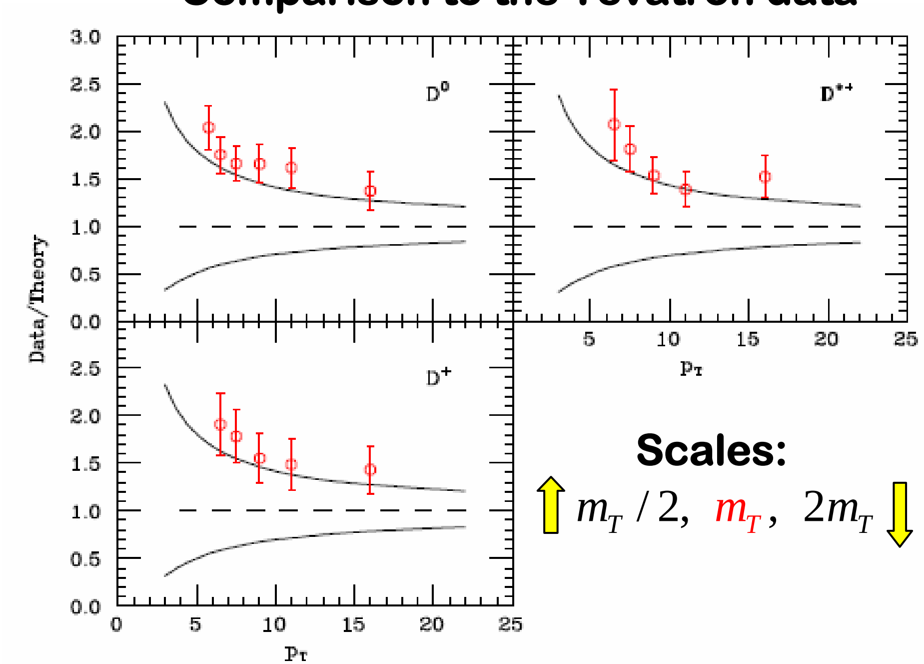


M.Cacciari, P.Nason, JHEP 9805 (1998)

- The new scale, mass, implies large logarithms, but …
- The contribution of logarithms is small in measurable p_T ranges
- The quarks are treated as “heavy” – in the fixed order calculation. Implies that NLO generates the PDF for charm and bottom (mostly)

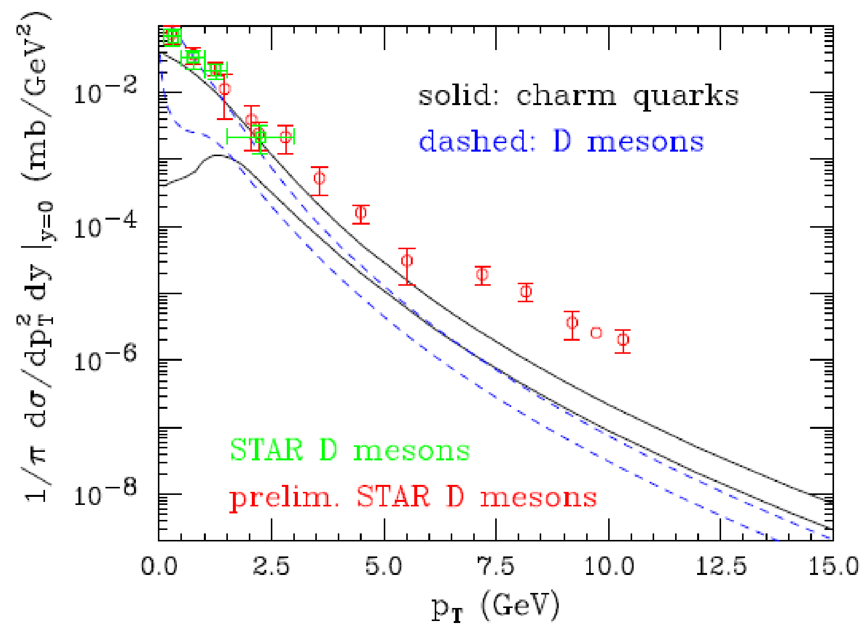
Phenomenological Results

Comparison to the Tevatron data



M.Cacciari, P.Nason, JHEP 0309 (2003)

Comparison to the RHIC data



R.Vogt et al., Phys.Rev.Lett.95 (2005)

Scales:

$m_T/2$, m_T, $2m_T$

$$\frac{\partial}{\partial\mu}\Big(\sigma = H(\alpha_s(\mu))\phi(\mu)\Big) = 0 \;\rightarrow \frac{\partial}{\partial\mu}\ln H(\alpha_s(\mu)) + \frac{\partial}{\partial\mu}\ln\phi(\mu)$$

- Description of open charm at the Tevatron is within uncertainties but not perfect
- Residual large scale uncertainties – should be careful with consistent choices
- At RHIC perturbative calculations under predict the data by factor of 2 – 4. Whether it is experimental systematic, incomplete theory or both – open question

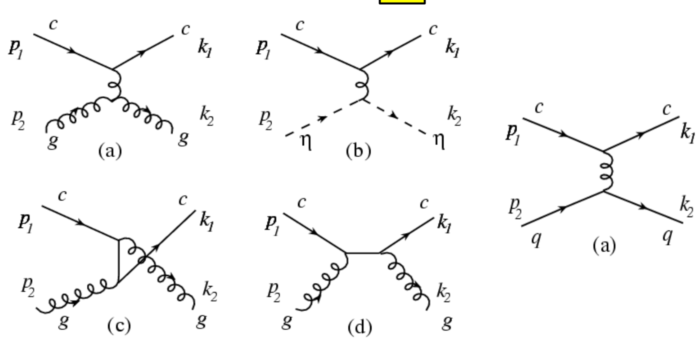
Detailed Analysis to LO

Single inclusive D - mesons | **D - meson triggered back-to-back correlations**

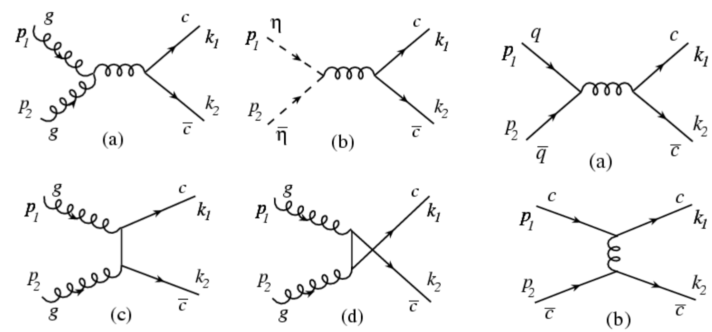
$$\frac{d\sigma_{NN}^{D_1}}{dy_1 d^2p_{T_1}} = K_{NLO} \sum_{abcd} \int_{x_{1,2}\leq 1} dy_2 \int_{x_{1,2}\leq 1} dz_1 \times \frac{1}{z_1^2} D_{D_1/c}(z_1) \frac{\phi_{a/N}(x_a)\phi_{b/N}(x_b)}{x_a x_b} \frac{\alpha_s^2}{S^2} |\overline{M}_{ab\to cd}|^2$$

$$\frac{d\sigma_{NN}^{D_1 h_2}}{dy_1 dy_2 dp_{T_1} dp_{T_2}} = K_{NLO} \sum_{abcd} 2\pi \int_D \frac{dz_1}{z_1} D_{D_1/c}(z_1) D_{h_2/d}(z_2) \times \frac{\phi_{a/N}(x_a)\phi_{b/N}(x_b)}{x_a x_b} \frac{\alpha_s^2}{S^2} |\overline{M}_{ab\to cd}|^2$$

Flavor excitation



Flavor creation



Faster convergence of the perturbative series

$$\left\langle |M|^2 \right\rangle \sim 2(LN)^2$$

$$LN \sim 10,100,1000...$$

F.Olness et al., Phys.Rev.D59 (1999)

Two different expansions

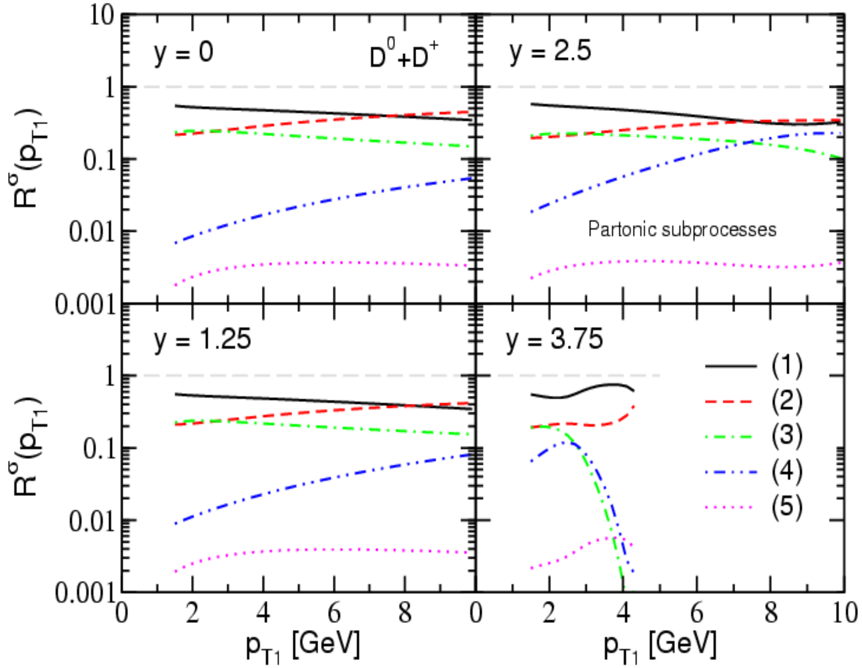
Over most of phase space

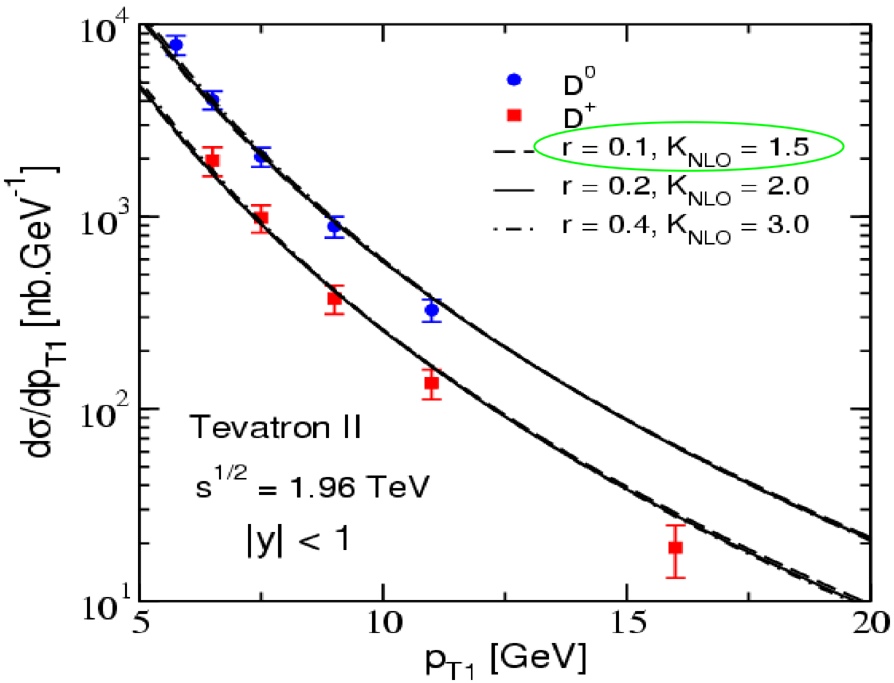
$$\left|\frac{\hat{t}}{\hat{s}}\right| \approx \left|\frac{\hat{t}}{\hat{u}}\right| = 1/LN \ll 1, \text{ or } \left|\frac{\hat{u}}{\hat{s}}\right| \approx \left|\frac{\hat{u}}{\hat{t}}\right| = 1/LN \ll 1$$

Slower convergence of the perturbative series

$$\left\langle |M|^2 \right\rangle \sim \frac{1}{6}(LN)$$

Numerical Results and Partonic Sub-Processes





Partonic sub-processes

$$(1)\ cg \to cg,\ (2)\ cq(\bar{q}) \to cq(\bar{q})$$
$$(3)\ gg \to c\bar{c},\ (4)\ q\bar{q} \to c\bar{c}$$
$$(5)\ c\bar{c} \to c\bar{c}$$

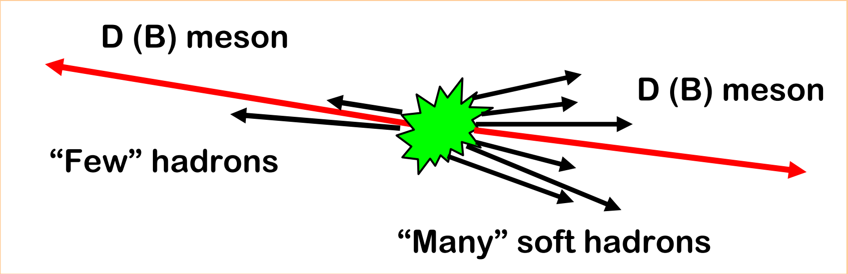
$$R^\sigma(p_{T_1}) = \frac{d\sigma^{D_1}_{ab\to cd}}{dy_1 d^2 p_{T_1}} \Big/ \frac{d\sigma^{D_1}_{tot}}{dy_1 d^2 p_{T_1}}$$

FFs: Braaten et al., Phys.Rev.D51 (1995)
PDFs: CTEQ 6.1 LO, J.Pumplin et al., JHEP 207 (2002)

- Meaningful K-factors (otherwise K>4)
- Anti-correlation between K and the hardness of fragmentation r
- If (LO,c-PDF) ~(NLO,no c-PDF) what are the corrections from (NLO,c-PDF)?

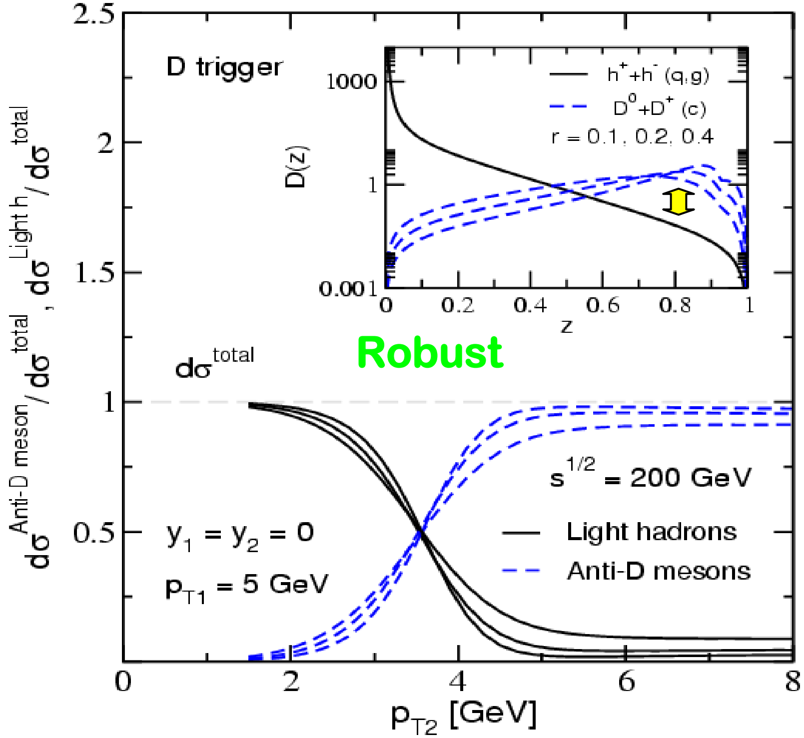
Hadron Composition of C (B) Triggered Jets

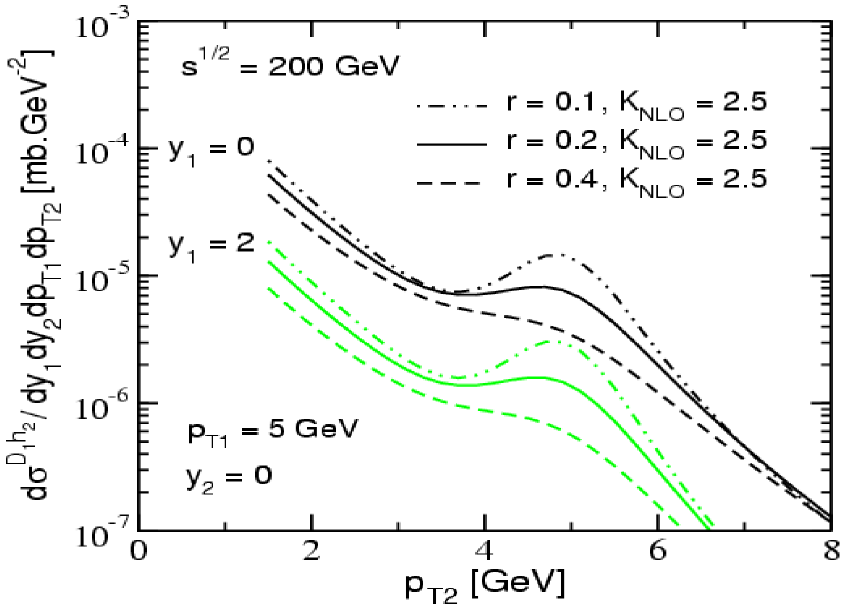
Possibility for new measurements of heavy flavor production at RHIC



$$\sim D_{D/c}(z,Q^2)/D_{h/q,g}(z,Q^2)$$

- Can clarify the underlying hard scattering processes and open charm production mechanisms
- Can constrain the hardness of D and B meson fragmentation

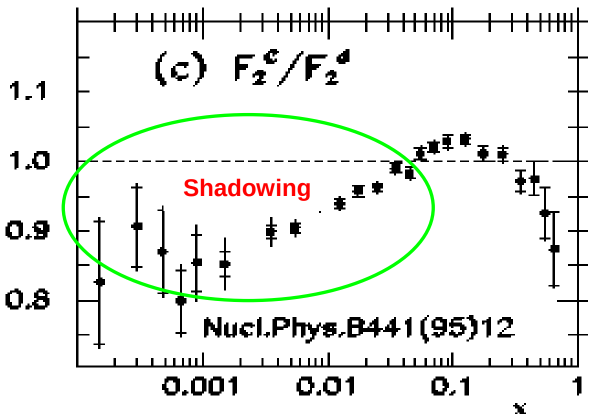




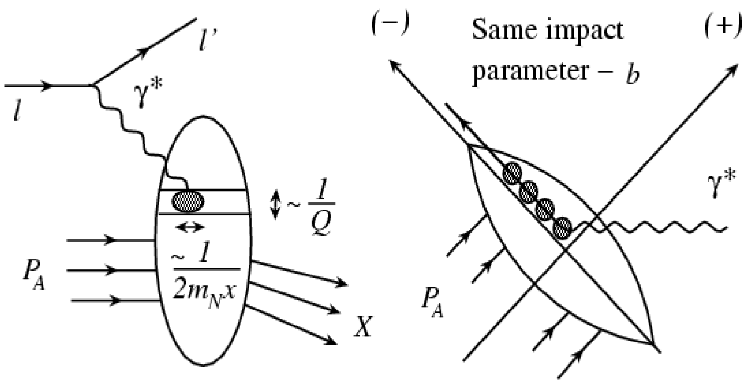
Cold Nuclear Matter Effects (I)

Calculating dynamical shadowing for heavy quarks

Shadowing: $\sigma_A \neq A \times \sigma_p$



Physics: uncertainty principle, i.e. coherence



"Mainstream" approach

$$F_T(x,Q^2) = \frac{1}{2}\sum_f Q_f^2 \int d\lambda_0 e^{ix\lambda} \left\langle p \left| \bar{\Psi}(0) \frac{\gamma^+}{2p^+} \Psi(\lambda_0) \right| p \right\rangle$$
$$= \frac{1}{2}\sum_f Q_f^2 \, \phi_f(x,Q^2) + \mathcal{O}(\alpha_s)$$

- Holds only to lowest order and leading twist
- Ignores multiple scattering

Longitudinal size: $\sim 1/2m_N x$

If $x < 0.1$ then $\Delta z > r_0$

Transverse size: $\sim 1/Q$

If $Q < m_N$ then exceed the parton size

What remains for theory: demonstrate that FS power corrections lead to suppression

The Idea Behind the Calculation

- **Lightcone gauge:** $A\cdot n = A^{+} = 0$
- **Breit frame:** $\bar{n} = [1, 0, 0_{\perp}], \ n = [0, 1, 0_{\perp}]$

$$q = -xp^{+}\bar{n} + \frac{Q^2}{2xp^{+}} n, \ \ p = \bar{n}p^{+}, \ \ xp + q = \frac{Q^2}{2xp^{+}} n$$

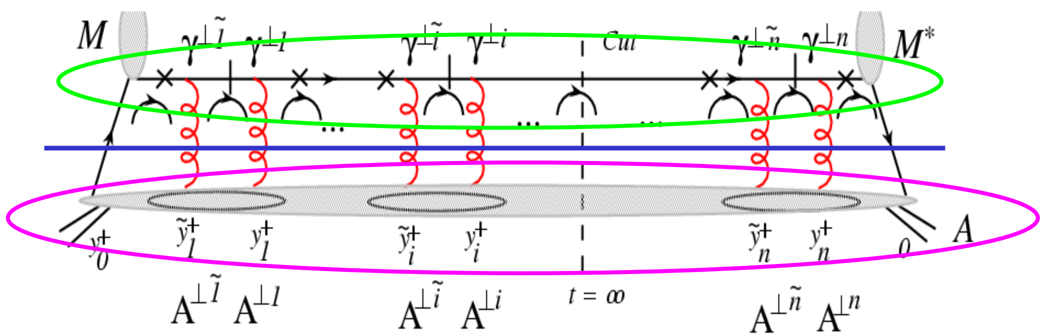
$$Cut = (2\pi)\frac{\gamma^{+}}{2p^{+}}\delta(x_i - x_B)$$

$$\Delta(x_i p + q) = \pm i \frac{\gamma^{+}}{2p^{+}} \frac{1}{x_i - x \pm i\varepsilon} \pm i \frac{xp^{+}\gamma^{-}}{Q^2}$$

Long distance **Short distance**

Perturbative

Hard part



Non-perturbative

Matrix element

$$\langle P | \hat{O}^{T=2+2n} | P \rangle = A \langle P/A | \hat{O}_q^{\ T=2} | P/A \rangle$$

Decompose $\prod_{i=1}^{n} \langle P/A | \hat{O}_g^{\ T=2} | P/A \rangle$

Contribution of single scatter: $\sim \xi^2 / Q^2$

$$\xi^2 = \left(\frac{3\pi\alpha_s(Q^2)}{8 r_{0\perp}^2} \right) \langle p | \hat{F}^2(\lambda_i) | p \rangle$$

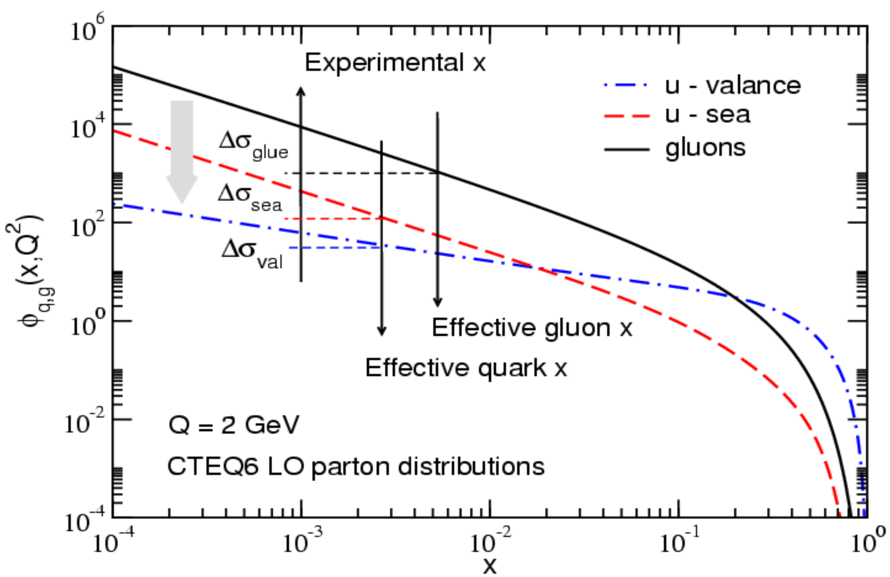
$$\hat{F}^2(\lambda_i) = \int \frac{d\tilde{\lambda}_i}{2\pi} \frac{F^{+\alpha}(\lambda_i) F_{\alpha}^{\ +}(\tilde{\lambda}_i)}{(p^{+})^2} \theta(\lambda_i - \tilde{\lambda}_i) \Rightarrow \lim_{x\to 0} \frac{1}{2} xG(x, Q^2)$$

High Twist Shadowing in DIS

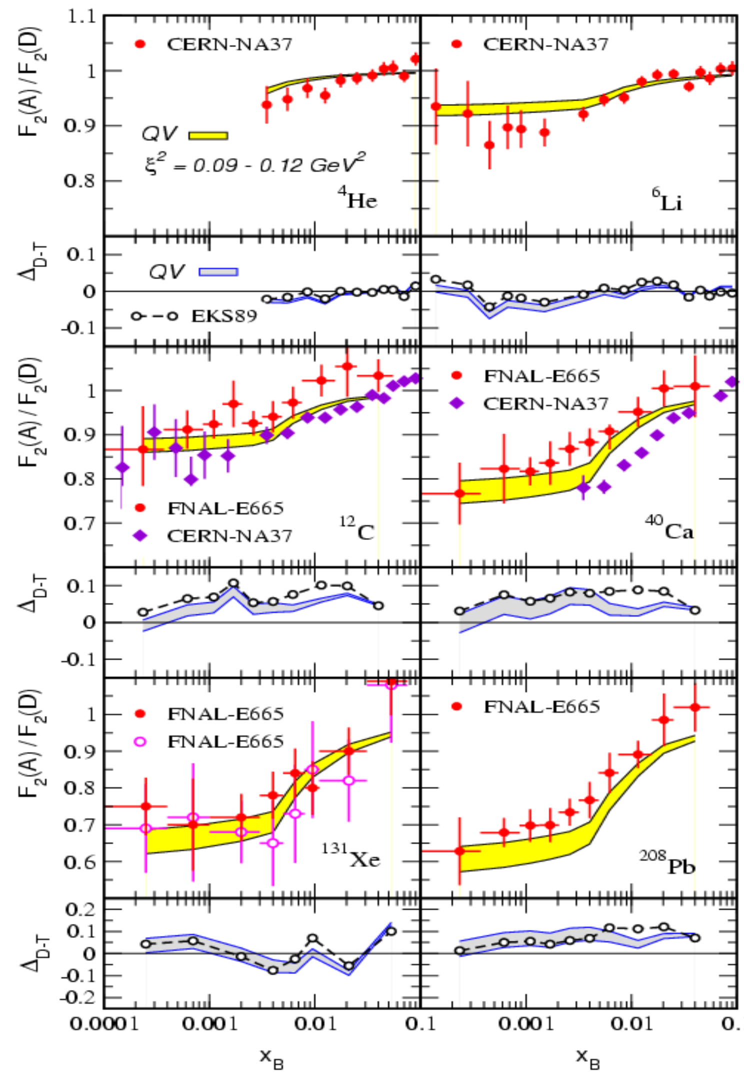
$$F_T^A(x,Q^2) \approx A\, F_T^{(LT)}\left(x + \frac{x\xi^2(A^{1/3}-1)}{Q^2}, Q^2\right) = A\, F_T^{(LT)}\left(x\left(1+\frac{m_{dyn}^2}{Q^2}\right), Q^2\right)$$

x = energy = mass

- Dynamical parton mass (QED analogy): $m_{dyn}^2 = \xi^2 A^{1/3}$



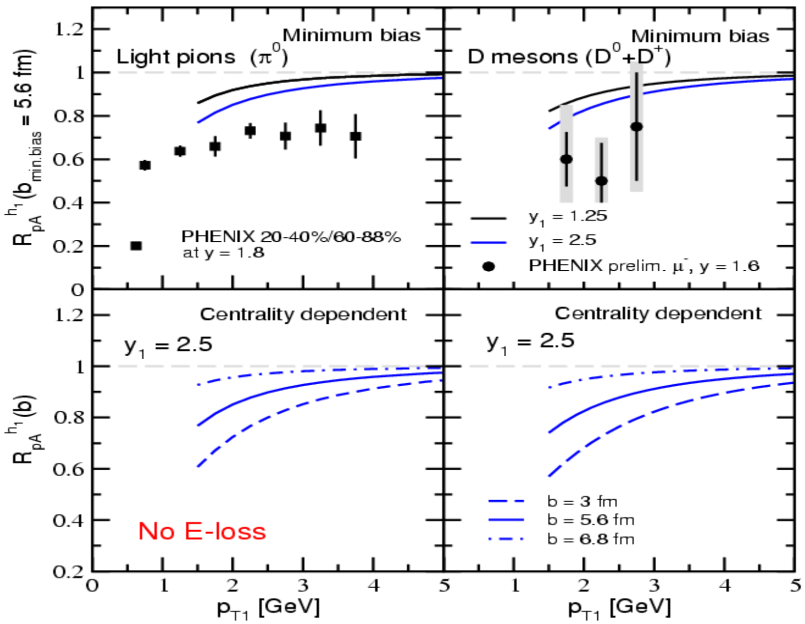
Can predict dynamical suppression effects on sea q, valence q and g



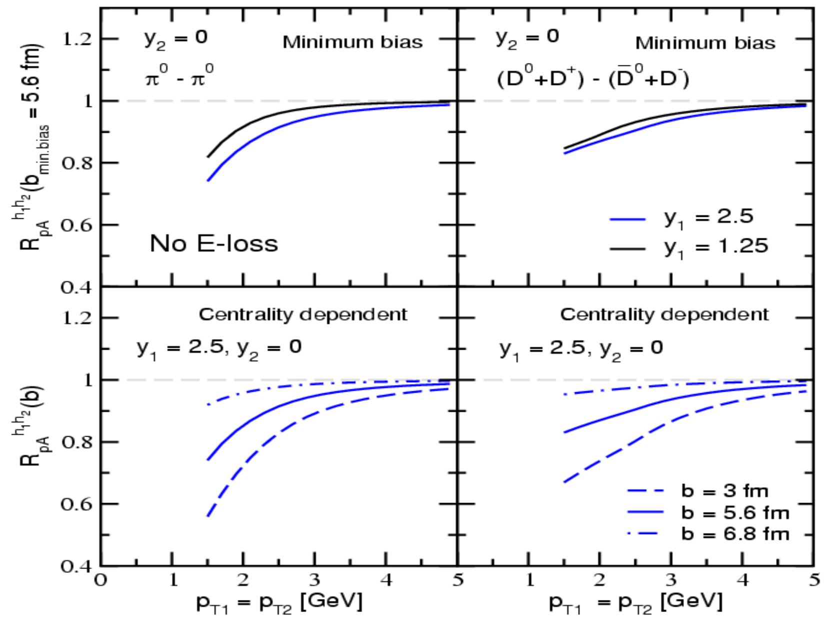
J.W.Qiu, I.V., Phys.Rev.Lett. 93 (2004)

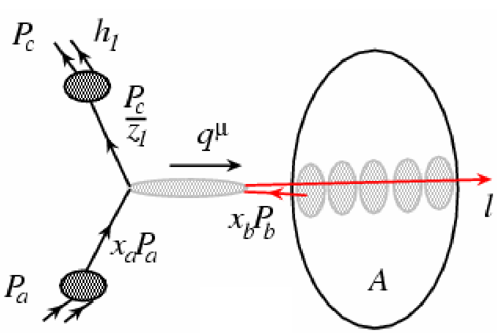
HTS for Light Hadrons and Open Charm

Single inclusive particles



Back-to-back correlations





$$F(x_b) = \frac{\phi(x_b)}{x_b} \left| \bar{M}^{\,2}_{ab \to cd} \right|$$

$$F(x_b) \;\to\; F\left(x_b + x_b C_d \, \frac{\xi^2}{-t + m_d^2} \, (A^{1/3} - 1) \right)$$

J.W.Qiu, I.V., Phys.Lett.B632 (2006)

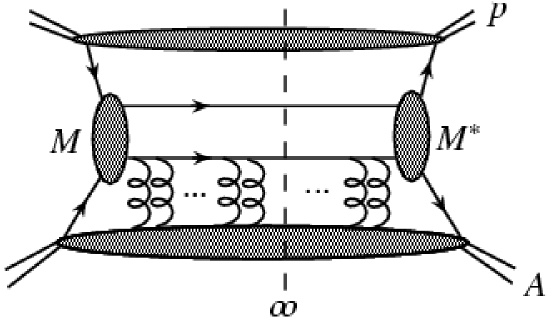
- Very similar dynamical shadowing for light hadrons and heavy quarks

$$S_{HT} = S_{HT}(q(g); \hat{t}(z_1, (z_2)))$$

- Insufficient to explain the forward rapidity data
- Single and double inclusive cross sections are similarly suppressed

Process Dependence of Power Corrections

Suppression ($\hat{t} < 0$) (For example forward rapidity)

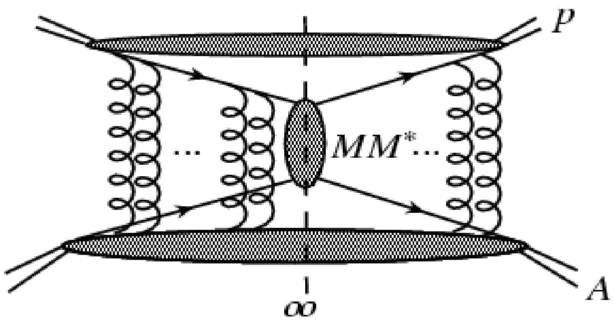


$$F_{ab\to cd}(x_b) \Rightarrow \sum_{n=0}^{\infty} \frac{1}{n!}\left(\tilde{x}_b \frac{\xi^2(B^{1/3}-1)}{-\hat{t}}\right)^n \frac{d^n}{dx_b^n} F_{ab\to cd}(x_b) = \exp\left[\tilde{x}_b \frac{\xi^2(B^{1/3}-1)}{-\hat{t}} \frac{d}{dx_b}\right] F_{ab\to cd}(x_b)$$

$$= F_{ab\to cd}\left(x_b + \tilde{x}_b C_d \frac{\xi^2(B^{1/3}-1)}{-\hat{t}}\right) = F_{ab\to cd}\left(x_b\left[1 + C_d \frac{\xi^2(B^{1/3}-1)}{-\hat{t}+m_d^2}\right]\right) .$$

- The function $F(x_b)$ contains the small x_b dependence

Enhancement ($\hat{s} > 0$) (For example DY)



$$F_{ab\to cd}(x_b) \Rightarrow F_{ab\to cd}\left(x_b\left[1 + C_a \frac{\xi^2(B^{1/3}-1)}{-\hat{s}+m_a^2}\right]\right)$$

- Power corrections are process dependent and not separable in PDFs and FFs

S.Brodsky et al, Phys.Rev.D65 (2002)

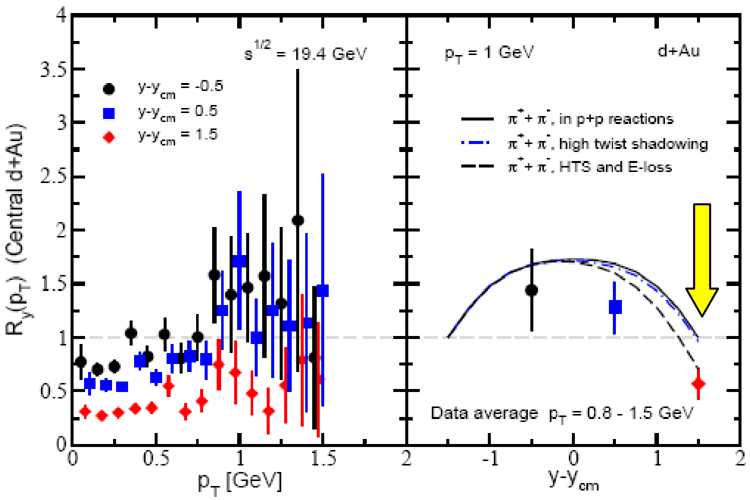
- Similar process dependence in single spin asymmetries

S.Brodsky et al, Phys.Lett.B530 (2002)

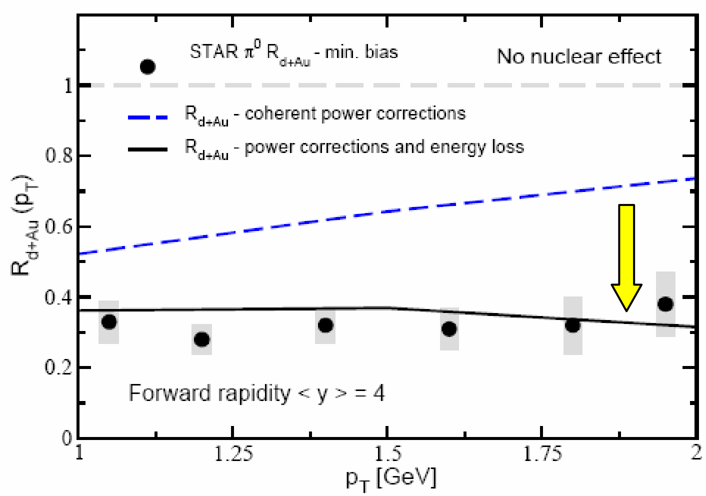
- Shadowing is dynamically generated in the hadronic collision

Cold Nuclear Matter Effects (II)

Implementing initial state energy loss



T.Alber et al., E.Phys.J.C 2 (1998)



S.S.Adler et al., nucl-ex/0603017

See also: **B.Kopeliovich, et al., Phys.Rev.C72 (2005)**

- Shadowing parameterizations: (NOT)

$$S_{LT} = S_{LT}(x, Q^2)$$

- Dynamical calculations of high twist shadowing: (NOT)

$$S_{HT} = S_{HT}(q(g); \hat{t}(z_1, (z_2)))$$

- Energy loss: in combination with HTS (YES)

Consistent application in all calculations

Initial state E-loss

$$\frac{dN_g^{(BG)}}{dy d^2k_\perp} = C_A \frac{\alpha_s}{\pi^2} \frac{q^2}{k_\perp^2 (k_\perp - q)^2} \qquad \phi(x, Q^2) \to \phi\left(\frac{x}{1-\varepsilon}, Q^2\right)$$

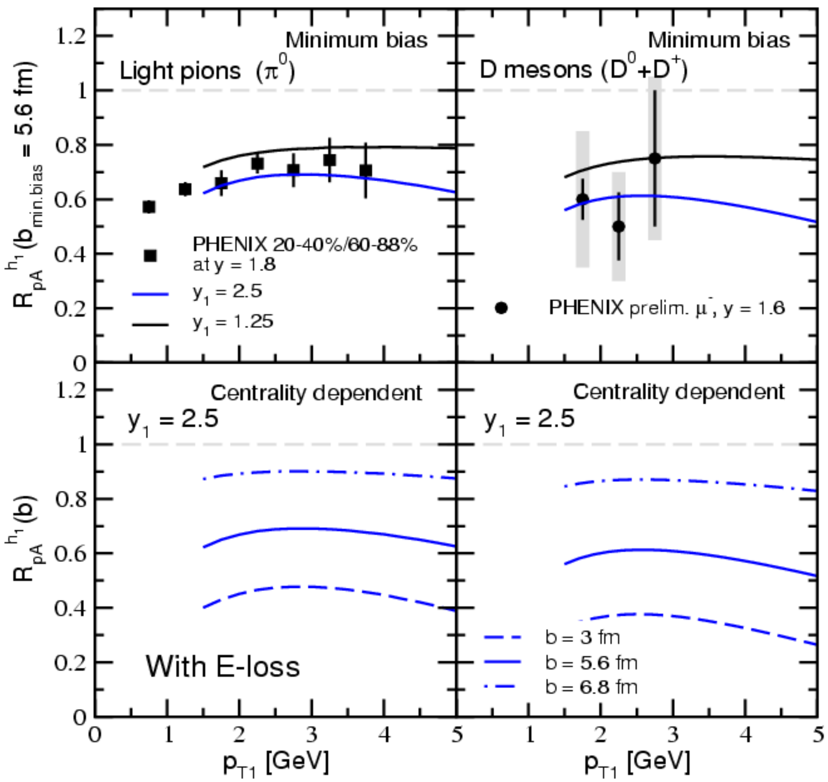
$$\varepsilon = \Delta E / E = k A^{1/3}, \;\; k_{\text{min bias}} = 0.0175$$

To investigate: $k = k(\mu, \lambda_{jet}, E, m)$

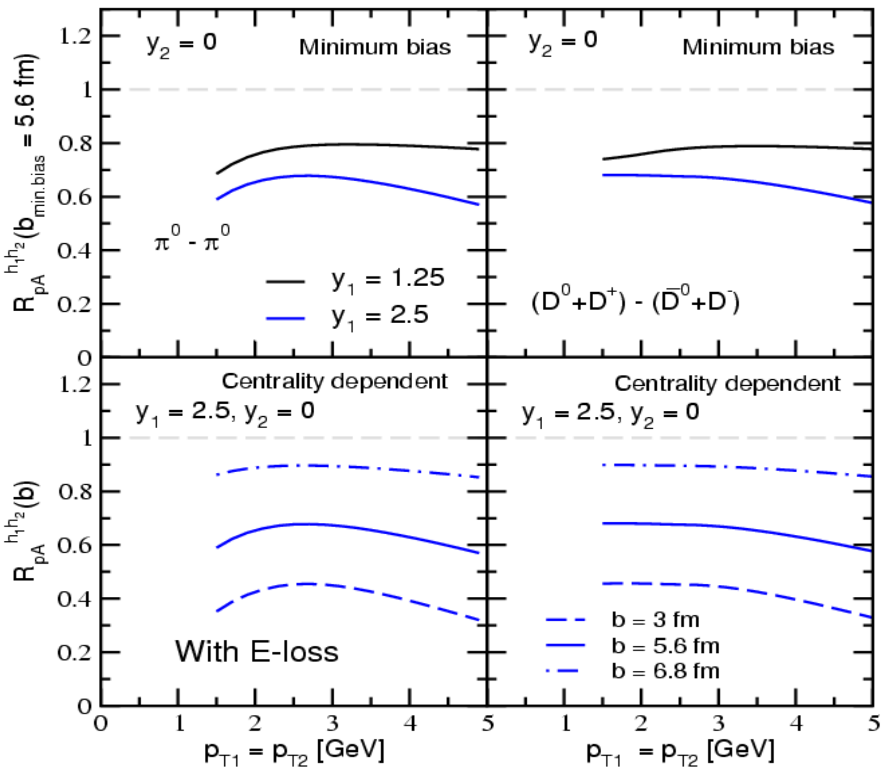
I.V., work in preparation

Energy Loss and High Twist Shadowing

Single inclusive particles



Back-to-back correlations



- **Main difference is much more p_T independent suppression as compared to high twist shadowing**

Same

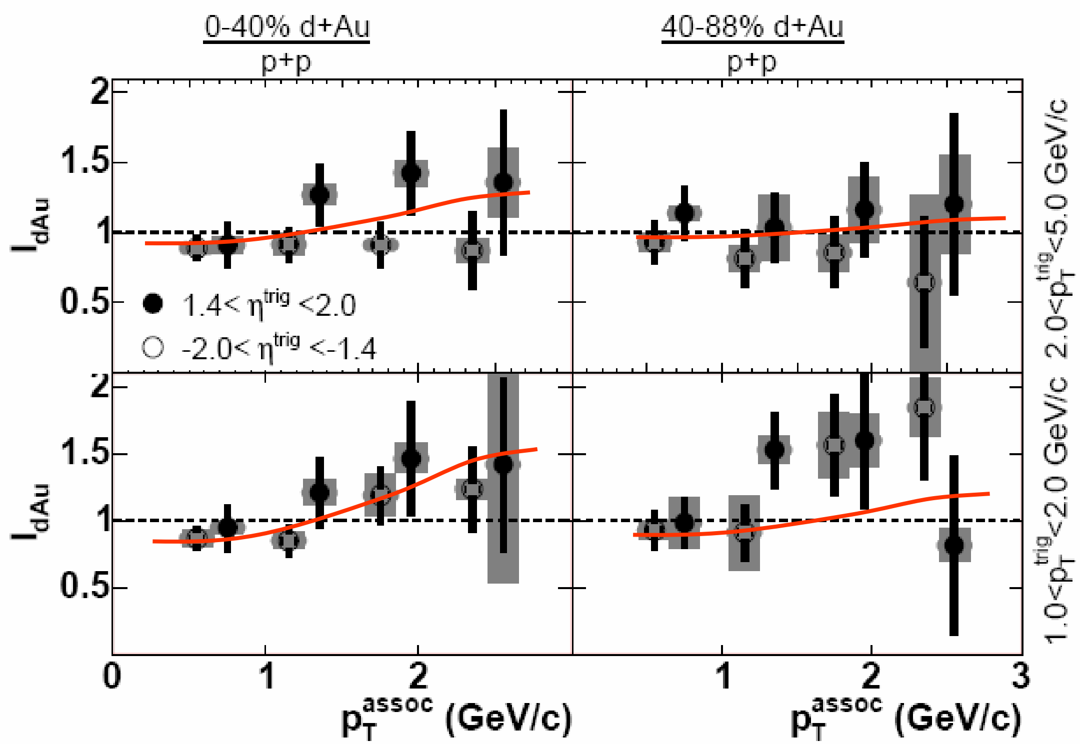
- Very similar e-loss effects for light hadron and heavy quark spectra
- Single and double inclusive cross sections are similarly suppressed

Constraints from Normalized Correlations

Why we considered dynamical shadowing and energy loss

Associated yield

$$I^{c}_{d+Au} = \frac{\dfrac{N^{d+Au}_{asso}}{N^{d+Au}_{trig}}}{\dfrac{N^{p+p}_{asso}}{N^{p+p}_{trig}}}$$



S.S.Adler et al., nucl-ex/0603017

Power corrections (high twist shadowing)

(forward rapidity, compare to solid symbols)

- Compatible with leading twist shadowing
- Compatible with high twist shadowing
- Compatible with IS energy loss
- Excludes large FS energy loss
- Excluded monojet phenomenology

PHENIX and STAR have put stringent constraints on pQCD models

Reaction Operator Approach

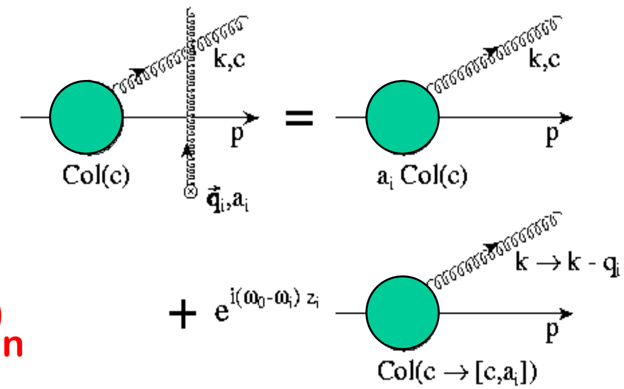
$$\omega_0 = \frac{\mathbf{k}^2}{2\omega}\,,\quad \omega_i = \frac{(\mathbf{k}-\mathbf{q}_i)^2}{2\omega}\,,\quad \omega_{(ij)} = \frac{(\mathbf{k}-\mathbf{q}_i-\mathbf{q}_j)^2}{2\omega}\,,\quad \omega_{(i\cdots j)} = \frac{(\mathbf{k}-\sum_{m=i}^{j}\mathbf{q}_m)^2}{2\omega}$$

$$E^+ \gg k^+ \gg \omega_{(i\cdots j)} \gg \frac{(\mathbf{p}+\mathbf{k})^2}{E^+}$$

$$\mathbf{H} = \frac{\mathbf{k}}{\mathbf{k}^2}\,,\qquad \mathbf{C}_{(i_1 i_2\cdots i_m)} = \frac{(\mathbf{k}-\mathbf{q}_{i_1}-\mathbf{q}_{i_2}-\cdots-\mathbf{q}_{i_m})}{(\mathbf{k}-\mathbf{q}_{i_1}-\mathbf{q}_{i_2}-\cdots-\mathbf{q}_{i_m})^2}\,,$$

$$\mathbf{B}_i = \mathbf{H} - \mathbf{C}_i\,,\qquad \mathbf{B}_{(i_1 i_2\cdots i_m)(j_1 j_2\cdots i_n)} = \mathbf{C}_{(i_1 i_2\cdots j_m)} - \mathbf{C}_{(j_1 j_2\cdots j_n)}\,.$$

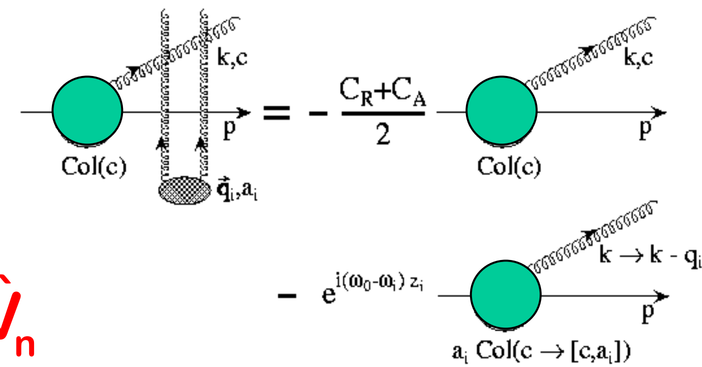
Single Born scattering



$\hat{D}_n$

$$-\left(-\frac{1}{2}\right)^{N_v} B_i\, e^{i\omega_0 z_i}\,[c,a_i]\,T_{el}$$

Double Born scattering



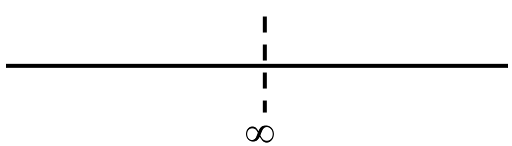
$\hat{V}_n$

$$-\frac{C_A}{2}\left(-\frac{1}{2}\right)^{N_v} B_i\, e^{i\omega_0 z_i}\, c\, T_{el}$$

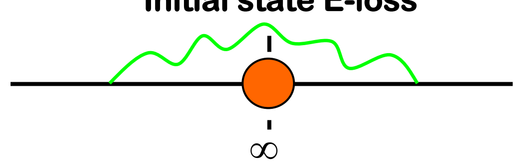
$$\hat{R} = D^\dagger D + V^\dagger + V = D^\dagger D - aD^\dagger - aD - (C_A - C_R) = \left(D^\dagger - a\right)\left(D - a\right) - C_A$$

Three types of initial conditions

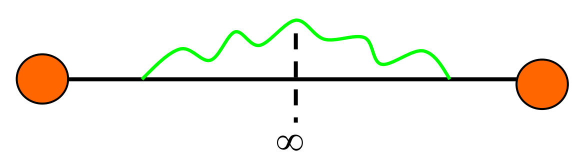
Bertsch-Gunion



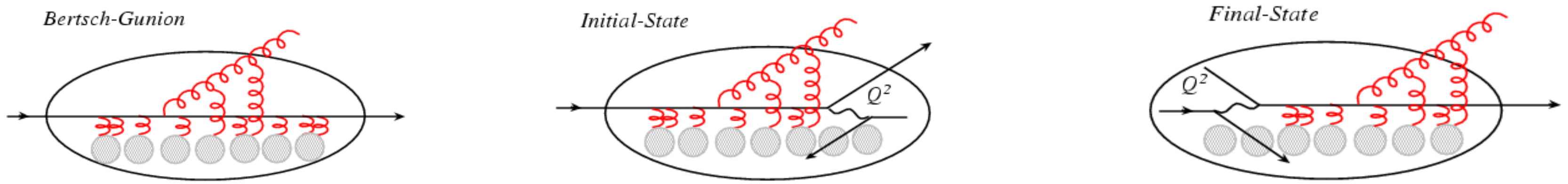
Initial state E-loss



Final state E-loss



Energy Loss to First Order in Opacity



Meaning of the expansion in "n"

- **Bertsch-Gunion Energy Loss**

$$\frac{\omega dN^g}{d\omega d^2k_\perp} = \frac{C_R\alpha_s}{\pi^2}\int_0^L \frac{d\Delta z}{\lambda_g}\int_0^{s/4} d^2q_\perp \frac{\mu_{eff}^2}{(q_\perp^2+\mu^2)^2}|B_1|^2$$

$$\boxed{\frac{\omega dN^g}{d\omega d^2k_\perp} = \frac{C_R\alpha_s}{\pi^2}\frac{L}{\lambda_g}\int_0^{s/4} d^2q_\perp \frac{\mu_{eff}^2}{(q_\perp^2+\mu^2)^2}\frac{q_\perp^2}{k_\perp^2(k_\perp-q_\perp)^2}}$$

- **Initial-State Energy Loss**

$$\frac{\omega dN^g}{d\omega d^2k_\perp} = \frac{C_R\alpha_s}{\pi^2}\int_0^L \frac{d\Delta z}{\lambda_g}\int_0^{s/4} d^2q_\perp \frac{\mu_{eff}^2}{(q_\perp^2+\mu^2)^2}\left[|B_1|^2 - 2H\cdot B_1\cos\frac{k_\perp^2\Delta z}{k^+}\right]$$

$$\boxed{\frac{\omega dN^g}{d\omega d^2k_\perp} = \frac{C_R\alpha_s}{\pi^2}\int_0^{s/4} d^2q_\perp \frac{\mu_{eff}^2}{(q_\perp^2+\mu^2)^2}\left[\frac{L}{\lambda_g}\frac{q_\perp^2}{k_\perp^2(k_\perp-q_\perp)^2} - 2\frac{q_\perp^2-2k_\perp\cdot q_\perp}{k_\perp^2(k_\perp-q_\perp)^2}\frac{k^+}{k_\perp^2\lambda_g}\sin\frac{k_\perp^2 L}{k^+}\right]}$$

- **Final-State Energy Loss**

$$\frac{\omega dN^g}{d\omega d^2k_\perp} = \frac{C_R\alpha_s}{\pi^2}\int_0^L \frac{d\Delta z}{\lambda_g}\int_0^{s/4} d^2q_\perp \frac{\mu_{eff}^2}{(q_\perp^2+\mu^2)^2}\left[-2C_1\cdot B_1\left(1\text{-}\cos\frac{(k_\perp\text{-}q_\perp)^2\Delta z}{k^+}\right)\right]$$

$$\boxed{\frac{\omega dN^g}{d\omega d^2k_\perp} = \frac{C_R\alpha_s}{\pi^2}\int_0^{s/4} d^2q_\perp \frac{\mu_{eff}^2}{(q_\perp^2+\mu^2)^2}\left[\frac{2k_\perp\cdot q_\perp}{k_\perp^2(k_\perp-q_\perp)^2}\left(\frac{L}{\lambda_g} - \frac{k^+}{(k_\perp\text{-}q_\perp)^2\lambda_g}\sin\frac{(k_\perp\text{-}q_\perp)^2 L}{k^+}\right)\right]}$$

New

Qualitatively

$$\frac{\Delta E}{E} \propto \frac{\sqrt{\mu^2}L}{\lambda_g}const(1)$$

$$\frac{\Delta E}{E} \propto \frac{\sqrt{\mu^2}L}{\lambda_g}const(2)$$

$$const(2) \ll const(1)$$

$$\frac{\Delta E}{E} \propto \frac{\mu^2 L^2}{\lambda_g}\frac{\ln E/E_0}{E}const$$

Numerical Results For Quark Energy Loss

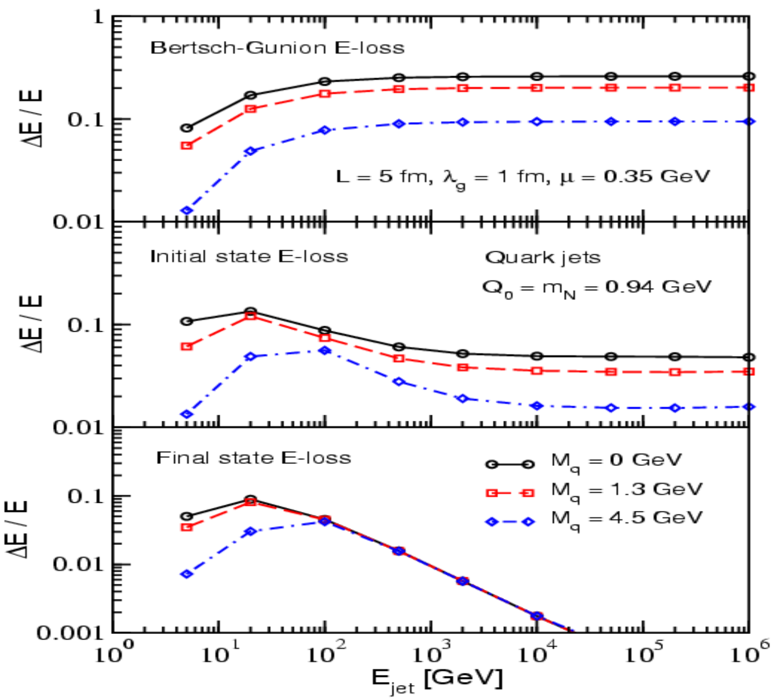
At any order in opacity we require $\sum_{i=1}^{n} \frac{dN^{g(i)}}{dyd^2k_\perp} > 0$

- Energetic quark jets can easily lose 20-30% of their energy, gluon jets x $C_A / C_F = 9/4$
- Coherence effects lead to cancellation of the medium-induced radiation

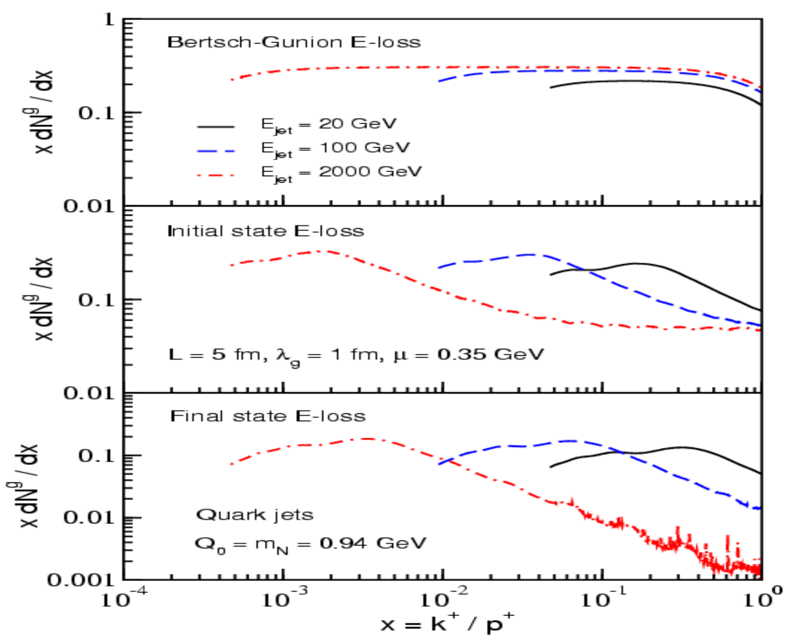
$$k_\perp^2 \to k_\perp^2 + Q_0^2 + x^2 M_q^2$$

- Initial state E-loss is much smaller than the incoherent Bertsch-Gunion limit
- Initial state E-loss is much larger than final state energy loss in cold nuclei

$x \to 1$ contribution to $x \frac{dN^g}{dx}$

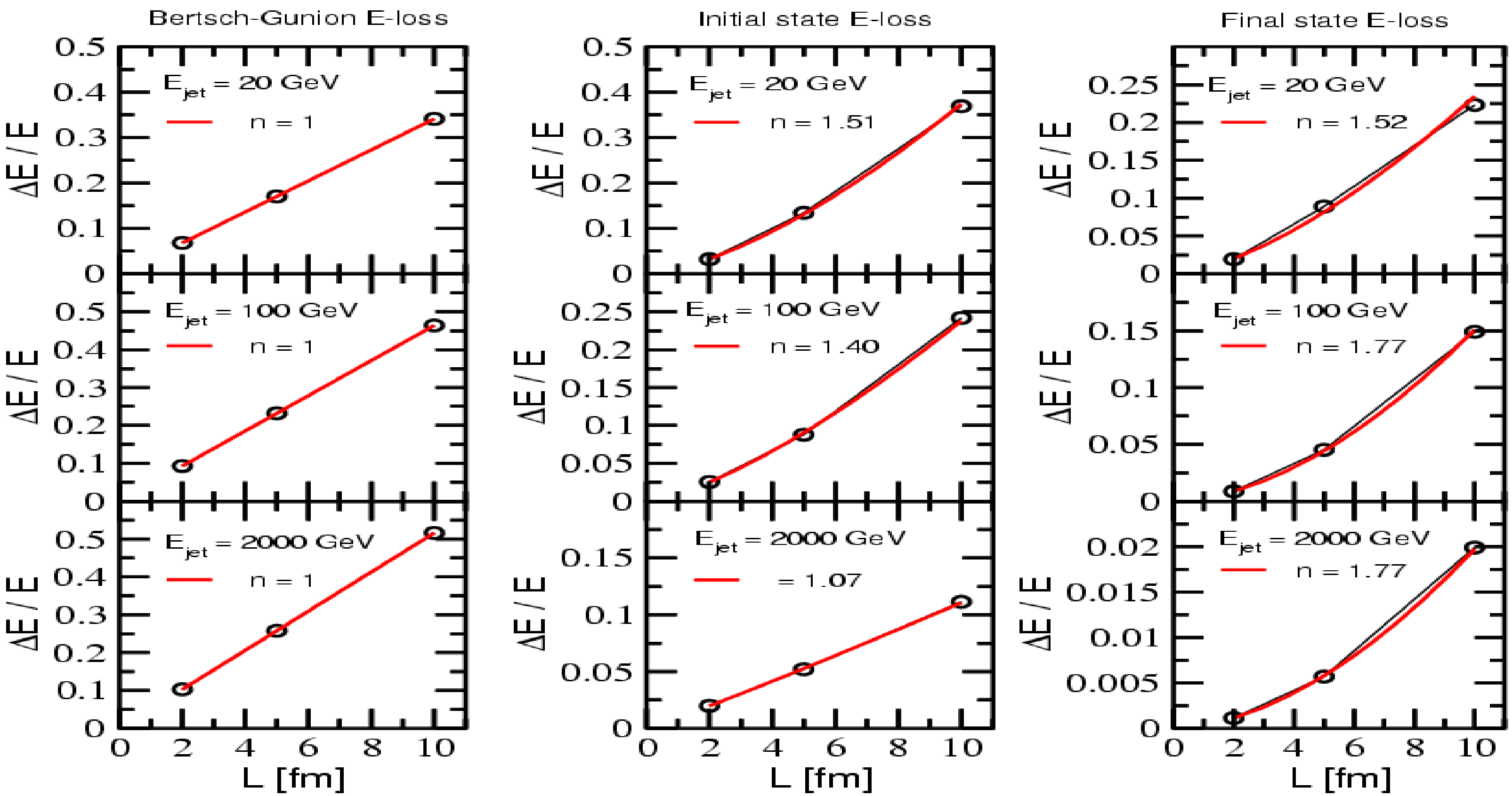


Fractional energy loss



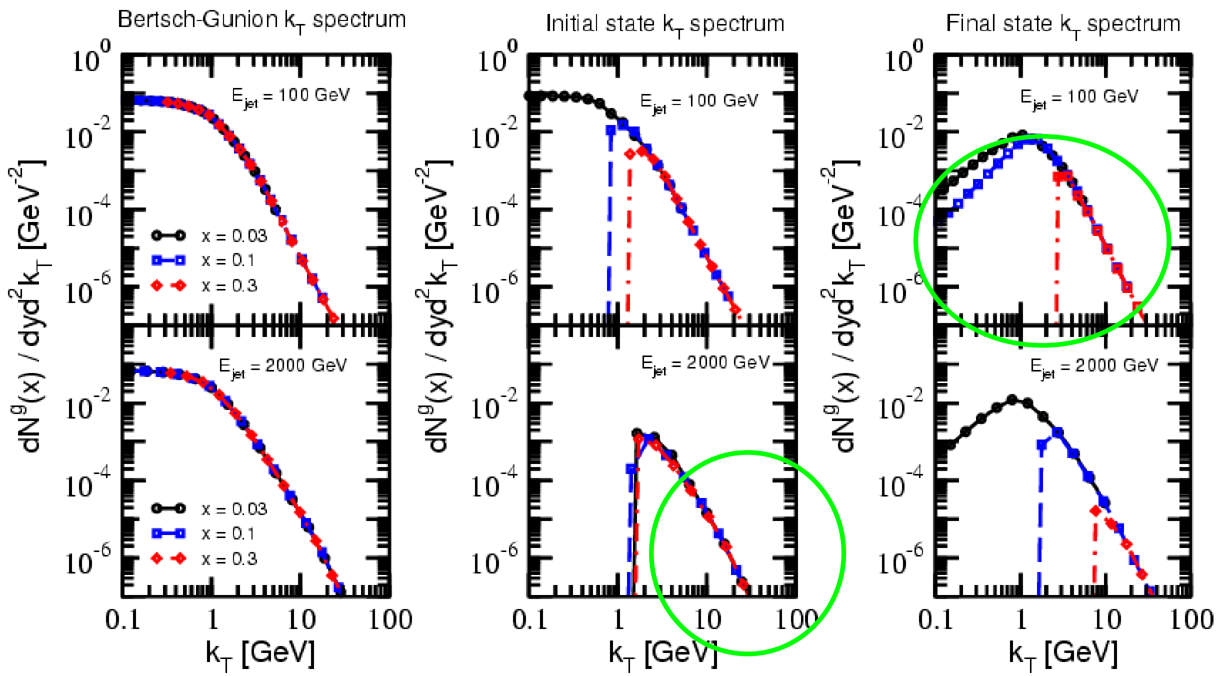
Radiation intensity

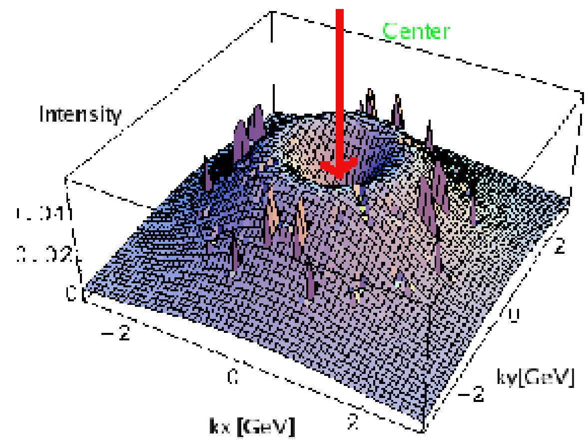
Path Length Dependence of E-Loss



- Bertsch-Gunion – linear dependence on L by definition
- Final state E-loss – approaches quadratic dependence on L, important for the centrality dependence and elliptic flow
- Initial state E-loss – approaches linear dependence on L, important for the centrality dependence in p+A reactions

Effects of Medium-Induced Radiation





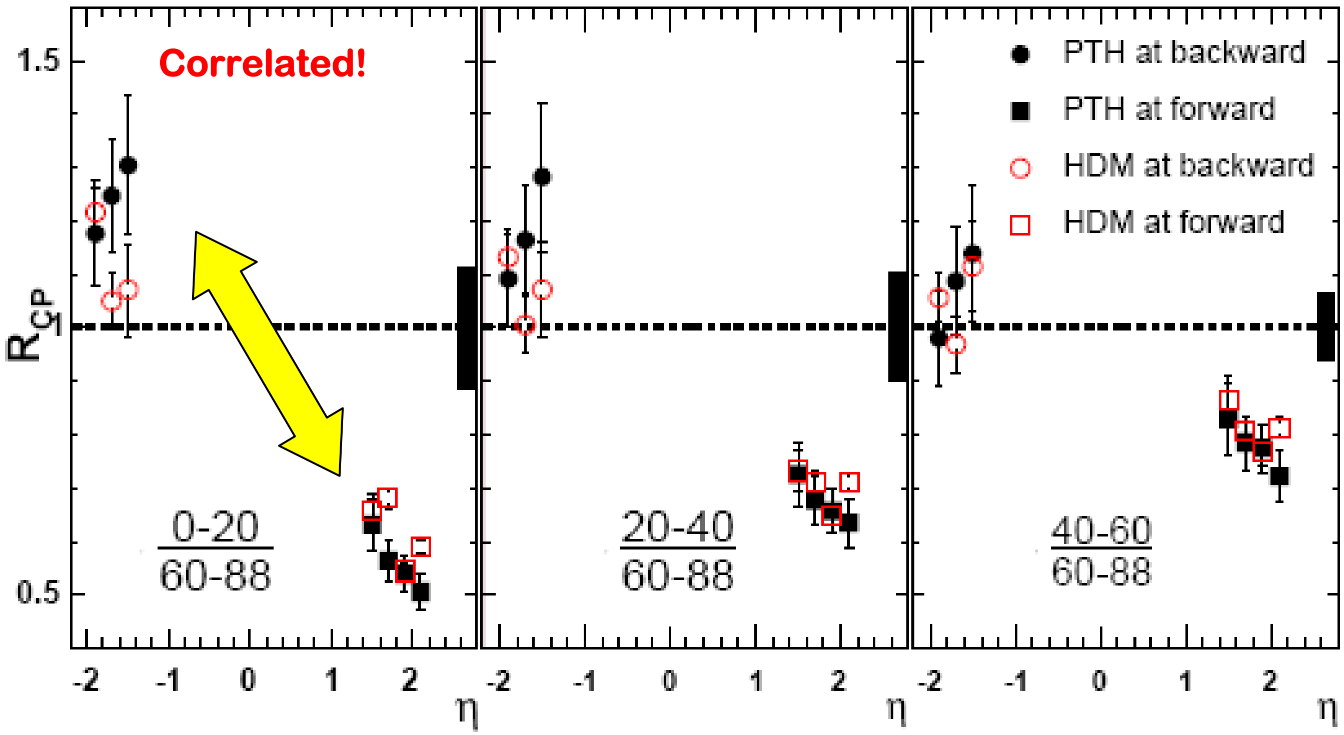
I.V., Phys.Lett.B630 (2005)

- **Cancellation of collinear radiation – large angle soft gluons and correspondingly soft hadrons**
- **Beyond the cancellation region - well defined power dependence**
- **The importance – hard scattering has the same power dependence**

$$\frac{dN^g}{dyd^2k_\perp} \sim \frac{1}{k_\perp^4}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{p_\perp^4}$$

Phenomenological Implications



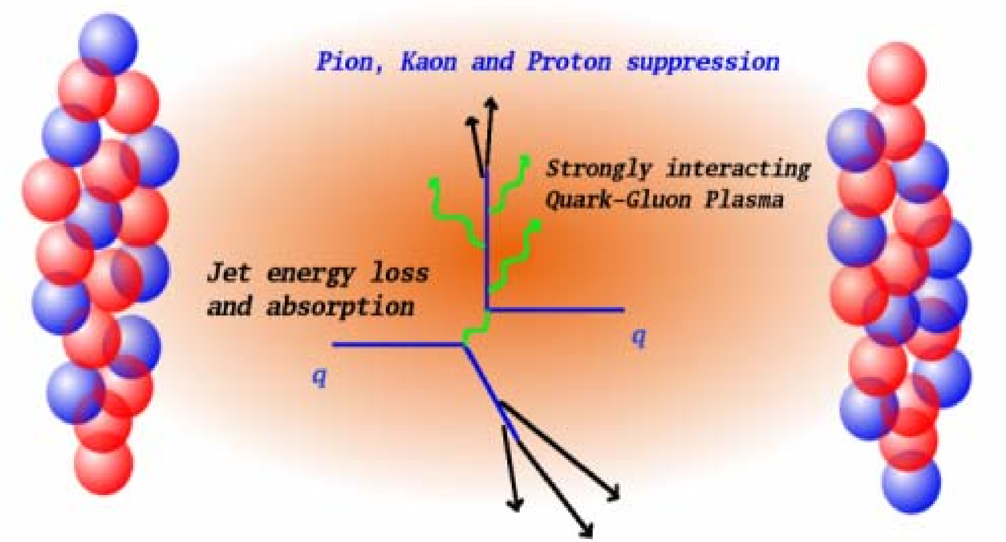
- **Suppression at forward rapidity – from energy loss of the incoming partons**
- **Enhancement at backward rapidity – comes from the redistribution of the lost energy**
- **Consistent pQCD code is still to be developed**

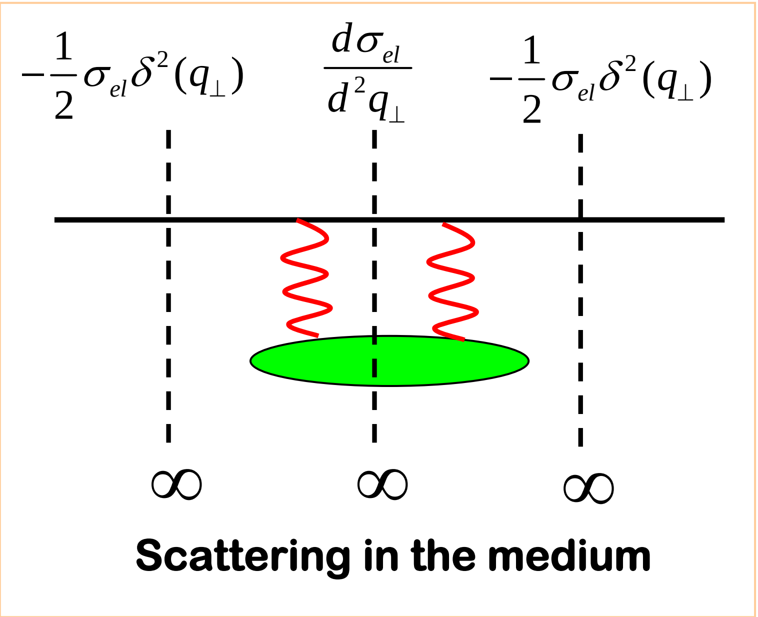
Conclusions

- **PQCD calculations of heavy quarks:**
  - There are open questions about heavy quark production at RHIC (even the Tevatron is not perfect)
  - Heavy quark triggered correlations provide a new possibility to constrain components of the PQCD calculations
- **High twist shadowing:**
  - Coherent final state scattering – good description of DIS
  - Generalized to p+A collisions and heavy quarks
  - Similar suppression of light hadrons and D mesons and inclusive spectra and correlations
  - Insufficient to explain the SPS & RHIC rapidity asymmetry
- **Non-Abelian energy loss in cold nuclei:**
  - Compatible with the world's data on hadron production in p+A reactions. Explains the forward rapidity suppression
  - Requires further investigation - being investigated now

In-Medium Modification of the PQCD Cross Sections

• The way to understand medium effects on hadron cross sections in the framework of PQCD is to follow the history of a parton from the IS nucleon wave function (PDF) to the FS hadron wave function (FF)





Range of the interaction in matter

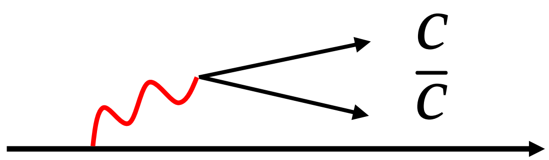
QGP: $\mu_D = g^2T^2\left(1+\frac{n_f}{6}\right) \qquad \lambda_D \sim \frac{1}{\mu_D}$

Cold nuclear matter: $r_0 \sim 1.2 \;\; fm$

$$\frac{d\sigma_{el}(R,T)}{d^2\mathbf{q}} = \frac{C_R C_2(T)}{d_A}\frac{|v(\mathbf{q})|^2}{(2\pi)^2} = \begin{Bmatrix} 2/9 \\ 1/2 \\ 9/8 \end{Bmatrix} \frac{4\alpha_s^2}{\left(q_\perp{}^2+\mu^2\right)^2}$$

Calculated in the Born approximation

Matrix Element Behavior



- Massless DGLAP evolution
- Matrix elements with charm mass created in the hard scatter

$$
\begin{aligned}
\langle|\mathcal{M}_{cg\to cg}|^2\rangle \;=\; & +\left\langle\frac{1}{2}\right\rangle \frac{g_s^4}{\hat{t}^2}\left(4\hat{t}^2-4\hat{s}\hat{u}-m_c^2\hat{s}-3m_c^2\hat{t}-m_c^4\right) \\
& -\left\langle\frac{2}{9}\right\rangle \frac{2g_s^4}{(\hat{u}-m_c^2)^2}\left(\hat{s}\hat{u}+2m_c^2\hat{u}-m_c^2\hat{s}\right)-\left\langle\frac{2}{9}\right\rangle \frac{2g_s^4}{(\hat{s}-m_c^2)^2}\left(\hat{s}\hat{u}+2m_c^2\hat{s}-m_c^2\hat{u}\right) \\
& -\left\langle-\frac{1}{4}\right\rangle \frac{2g_s^4}{\hat{t}(\hat{u}-m_c^2)}\left(2\hat{u}^2-5m_c^2\hat{u}+m_c^2\hat{t}-m_c^4\right)+\left\langle\frac{1}{4}\right\rangle \frac{2g_s^4}{\hat{t}(\hat{s}-m_c^2)}\left(2\hat{s}^2-5m_c^2\hat{s}+m_c^2\hat{t}-m_c^4\right) \\
& +\left\langle-\frac{1}{36}\right\rangle \frac{4g_s^4}{(\hat{s}-m_c^2)(\hat{u}-m_c^2)}\left(m_c^4-m_c^2\hat{t}\right)\;.
\end{aligned}
$$

$$
\langle|\mathcal{M}_{cq\to cq}|^2\rangle \;=\; \left\langle\frac{2}{9}\right\rangle \frac{2g_s^4}{\hat{t}^2}\left(\hat{s}^2+\hat{u}^2+m_c^2\hat{t}-m_c^4\right)
$$

$$
\begin{aligned}
\langle|\mathcal{M}_{gg\to c\bar{c}}|^2\rangle \;=\; & -\left\langle\frac{3}{16}\right\rangle \frac{4g_s^4}{\hat{s}^2}\left(\hat{s}^2-\hat{t}\hat{u}+m_c^2\hat{s}+m_c^4\right) \\
& +\left\langle\frac{1}{12}\right\rangle \frac{2g_s^4}{(\hat{t}-m_c^2)^2}\left(\hat{t}\hat{u}+m_c^2\hat{s}-2m_c^2\hat{t}-3m_c^4\right)+\left\langle\frac{1}{12}\right\rangle \frac{2g_s^4}{(\hat{u}-m_c^2)^2}\left(\hat{t}\hat{u}+m_c^2\hat{s}-2m_c^2\hat{u}-3m_c^4\right) \\
& -\left\langle\frac{3}{32}\right\rangle \frac{4g_s^4}{\hat{s}(\hat{t}-m_c^2)}\left(\hat{t}^2+m_c^2\hat{s}-2m_c^2\hat{t}+m_c^4\right)+\left\langle-\frac{3}{32}\right\rangle \frac{4g_s^4}{\hat{s}(\hat{u}-m_c^2)}\left(\hat{u}^2+m_c^2\hat{s}-2m_c^2\hat{u}+m_c^4\right) \\
& +\left\langle-\frac{1}{96}\right\rangle \frac{4g_s^4}{(\hat{t}-m_c^2)(\hat{u}-m_c^2)}\left(m_c^2\hat{s}-4m_c^4\right)\;.
\end{aligned}
$$

In most of phase space

$$\underbrace{\left|\frac{\hat{t}}{\hat{s}}\right| \approx \left|\frac{\hat{t}}{\hat{u}}\right| = 1/R \ll 1, \text{ or } \left|\frac{\hat{u}}{\hat{s}}\right| \approx \left|\frac{\hat{u}}{\hat{t}}\right| = 1/R \ll 1}$$

$$\sim 2R^2$$

$$\sim \frac{8}{9}R^2$$

versus

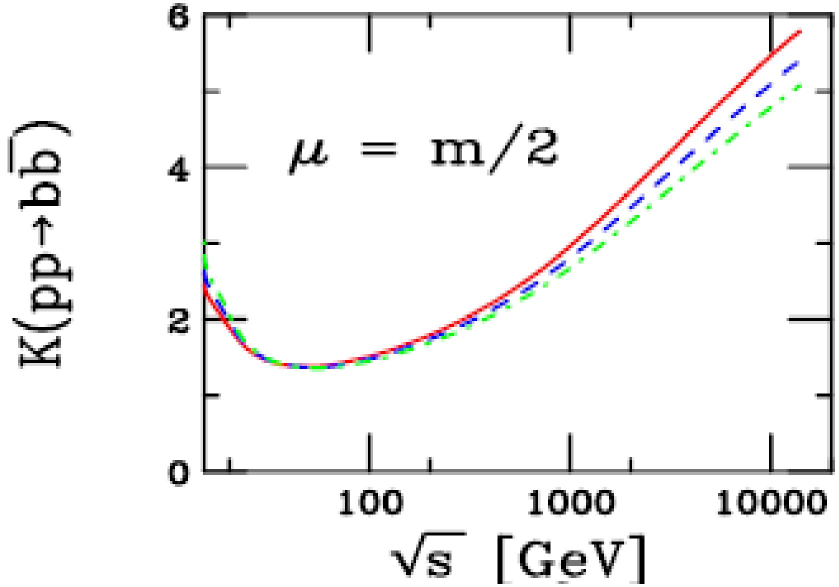
$$\sim \frac{1}{6}R$$

Kinematics and color can and do dominate over PDFs

Further Details on Perturbative Calculations

K - fctors

Kinematis of NLO calculations



$$s = (p_1 + p_2)^2 \,, \quad t = (p_1 - p)^2 \,, \quad u = (p_2 - p)^2 \,,$$

$$v = 1 + \frac{t}{s} \,, \quad w = \frac{-u}{s+t} \,.$$

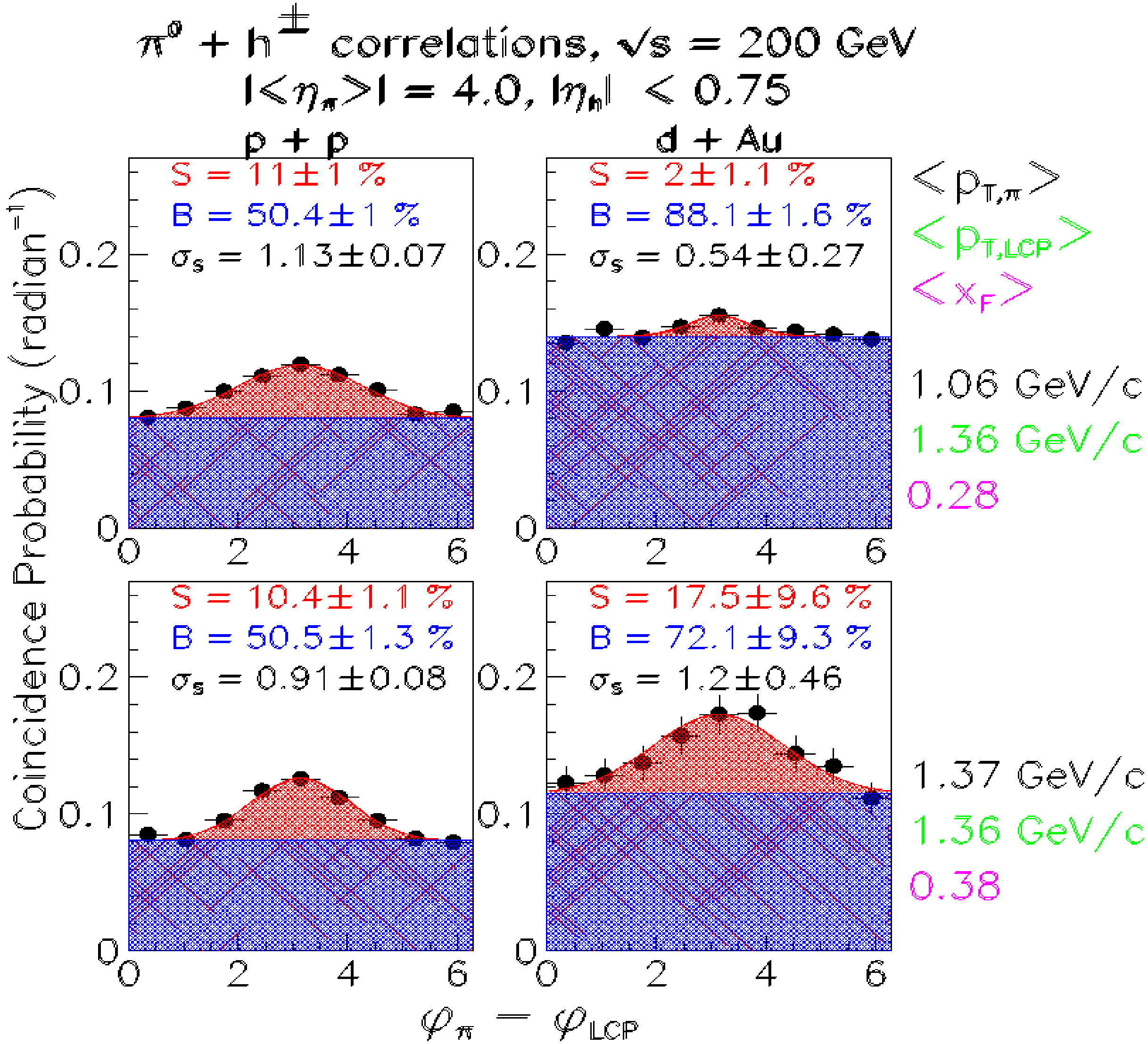
$$\frac{d^2\sigma}{dp_T^2 dy} = \frac{1}{S} \sum_{ijk} \int_{1-V+VW}^{1} \frac{dz}{z^2} \int_{VW/z}^{1-(1-V)/z} \frac{dv}{1-v} \int_{VW/zv}^{1} \frac{dw}{w} \times$$

$$\times F_{H_1}^{i}(x_1, \mu_F) F_{H_2}^{j}(x_2, \mu_F) D_k(z, \mu_F) \times$$

$$\times \left[\frac{1}{v} \left(\frac{d\sigma^0(s,v)}{dv} \right)_{ij \to k} \delta(1-w) + \frac{\alpha_s^3(\mu_R)}{2\pi} K_{ij \to k}(s, v, w; \mu_R, \mu_F) \right]$$

K-factors: R.Vogt, Heavy Ion Physics

Classification of PQCD Calculations



STAR data, updated now but qualitatively the same

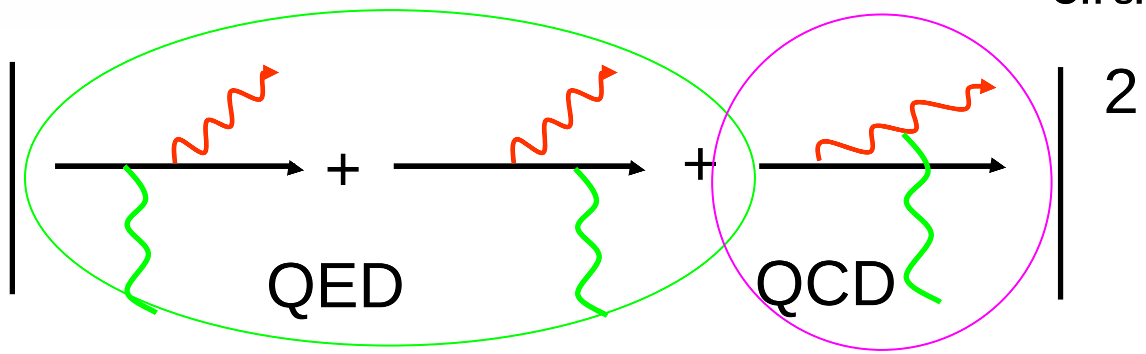
The width of the jet in d+Au is ½ the width in p+p. Largely looks like an experimental systematic: underestimating the yield and the width

E-loss Limits

G.Bertsch and F.Gunion

On shell weakly interacting quark

$$\mathcal{M}_1 \to -2ig_s\vec{\epsilon}_\perp \cdot \vec{B}_1 e^{it_1\omega_0}\left[c, a_1\right] = \mathcal{M}_{GB} \,,$$



$$\sim |B|^2$$

$$\mathcal{M}_1 = -2ig_s e^{it_0\omega_0}\vec{\epsilon}_\perp \cdot \left\{ \vec{H} a_1 c + \vec{B}_1 e^{it_{10}\omega_0}\left[c, a_1\right] + \vec{C}_1 e^{-it_{10}(\omega_1-\omega_0))}\left[c, a_1\right] \right\} .$$

Take the $t_0 \to -\infty$ **limit before squaring the amplitudes**

$$\frac{dN_g^{(GB)}}{dy\,d^2\vec{k}_\perp} = C_A \frac{\alpha_s}{\pi^2} \frac{q_1^2}{k_\perp^2 (k-q_1)_\perp^2}$$

Where $y = \ln 1/x$ **is interpreted as rapidity**

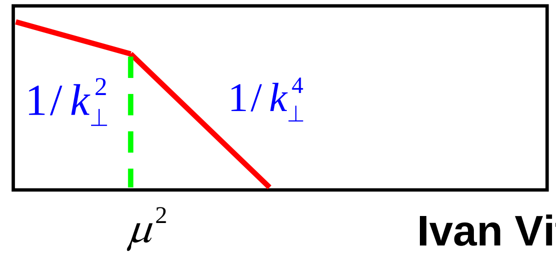
$$\frac{dN^g(BG)}{dy} \sim \frac{C_A \alpha_s}{\pi} \ln \frac{\mu^2}{\Lambda_{QCD}^2}$$

$$\frac{dN^g(QED)}{dy} \sim \frac{C_F \alpha_s}{\pi} \ln \frac{s}{\Lambda_{QCD}^2}$$

Argue the regulator (originally m_ρ**)**

$$\Delta E \sim E \frac{L}{\lambda} \frac{C_A \alpha_s}{\pi} \ln \frac{\mu^2}{\Lambda_{QCD}^2}$$

Can be large



Interplay of Dynamical and Physical Mass

The interplay of mass and kinematics

$$q^\mu = -\tilde{x}_b p_B^- n + \frac{-\hat{t}}{2\tilde{x}_b p_B^-}\bar{n} \qquad \delta((q+xp_B)^2 - m_d^2) \propto \delta(x - x_b) \qquad \Rightarrow \qquad \tilde{x}_b = \frac{x_b}{1 + m_d^2/(-\hat{t})}$$

Converting gluon fields to distributions

$$\int dx_0 \int \frac{d\tilde{y}_0^+}{2\pi}\, e^{ix_0^- p_B^- \tilde{y}_0^+} \left[\prod_{i=1}^{n} \int d\tilde{x}_i^-\, dx_i^- \int \frac{d\tilde{y}_i^+ p_B^-}{2\pi} \frac{dy_i^+ p_B^-}{2\pi}\, e^{i(\tilde{x}_i^- - x_{i-1}^-)p_B^- \tilde{y}_i^+} e^{i(x_i^- - \tilde{x}_i^-)p_B^- y_i^+} \right]$$
$$\times \left\langle P_B \middle| \mathcal{O}^{init} A^\perp(y_n^+) A^\perp(\tilde{y}_n^+) \cdots A^\perp(y_i^+) A^\perp(\tilde{y}_i^+) \cdots A^\perp(y_1^+) A^\perp(\tilde{y}_1^+) \middle| P_B \right\rangle$$
$$= \int dx_0 \int \frac{d\tilde{y}_0^+}{2\pi}\, e^{ix_0^- p_B^- \tilde{y}_0^+} \left[\prod_{i=1}^{n} \int d\tilde{x}_i^-\, dx_i^- \int \frac{d\tilde{y}_i^+}{2\pi} \frac{dy_i^+}{2\pi} \frac{e^{i(\tilde{x}_i^- - x_{i-1}^-)p_B^- \tilde{y}_i^+}}{i(\tilde{x}_i^- - x_{i-1}^- - i\epsilon)} \frac{e^{i(x_i^- - \tilde{x}_i^-)p_B^- y_i^+}}{i(x_i^- - \tilde{x}_i^- - i\epsilon)} \right]$$
$$\times \left\langle P_B \middle| \mathcal{O}^{init} F^{-\perp}(y_n^+) F_\perp^-(\tilde{y}_n^+) \cdots F^{-\perp}(y_i^+) F_\perp^-(\tilde{y}_i^+) \cdots F^{-\perp}(y_1^+) F_\perp^-(\tilde{y}_1^+) \middle| P_B \right\rangle$$

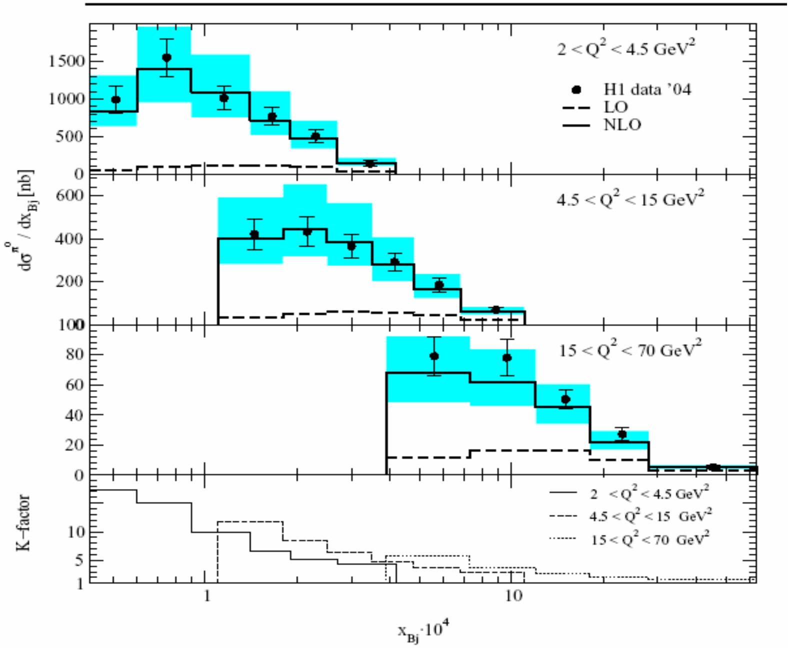
On shell cut

$$\mathcal{C}_{\mu\nu}(x_i, x_b, -\hat{t}, m_d) \;=\; 2\pi \left(\frac{\tilde{x}_b}{-\hat{t}} \right) (\tilde{p}\cdot\gamma + m_d)_{\mu\nu} \delta(x_i^- - x_b)$$

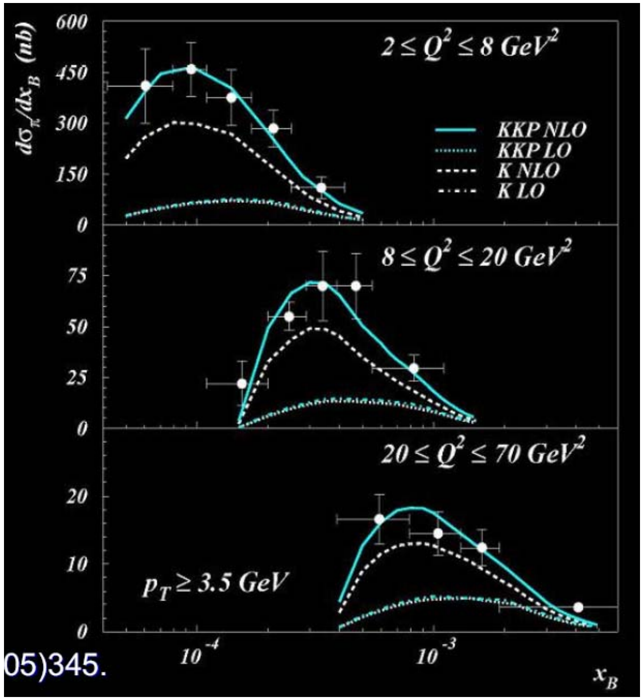
HERA Forward Jet Production

BFKL - enhanced cross sections, not suppressed

DIS 2005



$10^4 \times x_B$ ($p_T^* > 2.5$ GeV)



B.Khiehl

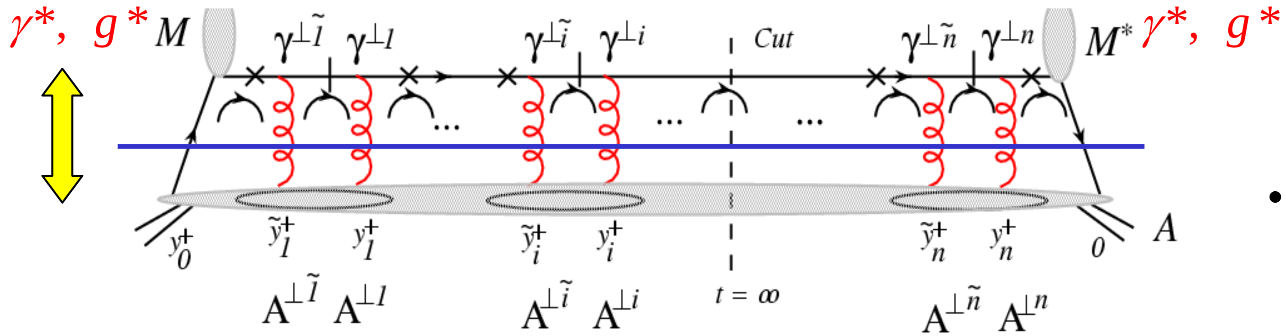
- DGLAP in good health (within present uncertainties)

– $\theta \to 0$ (or $\eta \to \infty$ or $x_F \to -1$): hadron h close to proton remnant $\rightsquigarrow$ fracture functions.

– $x_B \to 0$: BFKL dynamics. But no convincing case yet, see also forward-jet electroproduction (E. Gallo's talk).

R.Sassot

Resumming Final State Power Corrections



DIS or p+A or A+A reactions at small x

- Separate hard scattering from the high twist matrix elements (Fiertz ids.)

Two gluon paring is natural:

$$\Delta(x_i p + q) = \pm i \frac{\gamma^+}{2p^+} \frac{1}{x_i - x \pm i\varepsilon} \pm i \frac{xp^+\gamma^-}{Q^2}$$

$$\left\langle P_B \middle| \mathcal{O}^{init} \prod_{i=1}^{n} F^{-\perp}(y_i^+) F_\perp^-(\tilde{y}_i^+) \middle| P_B \right\rangle \approx B \langle p_B | \mathcal{O}^{init} | p_B \rangle \prod_{i=1}^{n} \frac{\rho(r)}{2E_{p_B}} \langle p_B | F^{-\perp}(y_i^+) F_\perp^-(\tilde{y}_i^+) | p_B \rangle$$

$$= B \langle p_B | \mathcal{O}^{init} | p_B \rangle \prod_{i=1}^{n} \frac{3}{8\pi r_0^3 m_N} \langle p_B | F^{-\perp}(y_i^+) F_\perp^-(\tilde{y}_i^+) | p_B \rangle$$

$$\xi^2 = \left(\frac{3\pi\alpha_s(Q^2)}{8r_{0\perp}^2} \right) \int \frac{dy^-}{2\pi} e^{i0p^+y^-} \left\langle p \middle| F^{+\perp} F_\perp^{\ +} \middle| p \right\rangle \theta(y^-) = \left(\frac{3\pi\alpha_s(Q^2)}{8r_{0\perp}^2} \right) \lim_{x\to 0} xG(x)$$

PHYSICS:

- QCD factorization approach, background color magnetic field
- Dynamical parton mass (QED analogy): $m^2_{dyn} = \xi^2 A^{1/3}$